Stress–energy tensor of neutral massive fields in the Reissner–Nordström spacetime

Jerzy Matyjasek*

Institute of Physics, Maria Curie-Skłodowska University,

pl. Marii Curie - Skłodowskiej 1,

20-031 Lublin, Poland

Abstract

The approximation of the renormalized stress-energy tensor of the quantized massive scalar, spinor, and vector field in the Reissner- Nordström spacetime is constructed. It is achieved by functional differentiation of the lowest order of the Schwinger-DeWitt effective action involving coincidence limit of the Hadamard-Minakshisundaram-DeWitt-Seely coefficient a_3, and restricting thus obtained general formulas to spacetimes with vanishing curvature scalar. For the massive scalar field with arbitrary curvature coupling our results reproduce those obtained previously by Anderson, Hiscock, and Samuel by means of 6-th order WKB approximation.

PACS numbers: 04.70.Dy, 04.62+v

UMCS-TG-99-11

---

*Electronic Address: matyjase@tytan.umcs.lublin.pl, jurek@iris.umcs.lublin.pl

I. INTRODUCTION

Treating the renormalized stress-energy tensor as the source term of the semiclassical Einstein field equations, one could, in principle, determine the back reaction of the quantized fields upon the spacetime geometry of a black hole unless the (unknown) quantum gravity effects become dominant. Mathematical difficulties encountered in the attempts to construct characteristics of the vacuum polarization in a concrete spacetime are well known and since the back reaction equations require knowledge of the functional dependence of stres-energy tensor of the quantized field, $\langle T^{\mu}_{\nu}\rangle_{ren}$, on a wide class of metrics, purely analytical treatment of the problem is impossible. It is natural therefore that much effort have been concentrated on developing approximate methods.

The vacuum polarization effects of the massive fields in the curved background has been studied by a number of authors [1-18]. It has been shown that for sufficiently massive fields (i.e. when the Compton length is much smaller than the characteristic radius of curvature, where the latter means any characteristic length scale of the geometry) the asymptotic expansion of the effective action in powers of m^{-2} may be used. It is because the nonlocal contribution to the total effective action can be neglected, and, consequently, the vacuum polarization part is local and determined by the geometry of the spacetime in question.

In the black hole spacetimes the vacuum polarization of the massive scalar, spinor, and vector fields have been constructed in a series of papers by Frolov and Zel'nikov in the vacuum type-D geometries [1-4]. They used general framework of the Schwinger - DeWitt method [12-18] and constructed the first order of the effective action, omitting the terms that do not contribute to a Ricci-flat spaces. Using a different method, Anderson, Hiscock, and Samuel evaluated the approximate $\langle T^{\mu}_{\nu}\rangle_{ren}$ of the massive scalar field with arbitrary curvature coupling for a general static, spherically symmetric spacetime and applied obtained formulas to the Reissner-Nordström spacetime [8]. Their approximation is equivalent to the Schwinger - DeWitt expansion; to obtain the lowest (i. e. m^{-2}) terms, one has to use sixth-order WKB approximation of the mode functions. Numerical calculations reported in

Ref.[8] indicate that the Schwinger-DeWitt method always provide a good approximation of the renormalized stress energy tensor of the massive scalar field with arbitrary curvature coupling as long as the mass of the field remains sufficiently large.

The aim of this paper is to construct the renormalized stress-energy tensor of the massive scalar with arbitrary curvature coupling, spinor, and vector fields in the geometry of the Reissner-Nordström black hole. To our knowledge the spinor and vector fields have not been discussed earlier. We shall achieve this using the standard result of the theory of quantized massive fields in the curved background that connects the coincidence limit of the HDSM (Hadamard-Minakshisundaram-DeWitt-Seely) coefficient $[a_3]$ with the lowest order of the one-loop effective action and consequently with the regularized stress-energy tensor [1-4, 19-23]. Indeed, functionally differentiating the effective action we obtain a general and rather complex expression for the renormalized stress-energy tensor that is valid in any spacetime. Then we specialize thus obtained formulas to the spacetimes with vanishing curvature scalar and apply the result to the Reissner- Nordström geometry. We show that for the scalar field the resulting $\langle T^{\mu}_{\nu}\rangle_{ren}$ is identical with the tensor obtained earlier by Anderson, Hiscock, and Samuel and that in the limit of vanishing electric charge it reduces to the stress-energy tensor constructed by Frolov and Zel'nikov.

There is another important limit of the general Reissner-Nordström geometry that yields the extremal black hole. Expanding the near-horizon region of such a geometry into a whole manifold one obtains the Bertotti-Robinson spacetime actively studied recently. We construct the stress-energy tensors in the Bertotti-Robinson spacetime taking appropriate limits in our general formulas and analyse the conditions under which this geometry is a self-consistent solution of the semiclassical Einstein field equations. Analyses carried out in the Bertotti-Robinson spacetime yield similar results.

The effective action approach that we employ in this paper requires the metric of the spacetime to be positively defined. Hence, to obtain the physical stress-energy tensors one has to analytically continue at the final stage of calculations their Euclidean counterparts. It should be stressed once again that the method, when applied to the rapidly varying or strong

gravitational fields, breaks down and that its massless limit is contaminated by nonphysical divergences.

II. THE EFFECTIVE ACTION

We begin with a short description of the method. More detailed presentation may be found in [3,4, 21-23]. Our notation corresponds to those of Refs. [21-23]. Consider the elliptic second-order differential operator of the form

$$F = g^{\mu\nu}\nabla_\mu\nabla_\nu + Q - m^2, \tag{1}$$

acting on the (super)field $\phi^A(x)$, where

$$Q = Q^A{}_B \tag{2}$$

is some matrix-valued function playing a role of the potential, ∇_μ is the appropriate covariant derivative and m^2 is a matrix satisfying $\nabla_\mu m^2$ and commuting with Q. It is unnecessary at this stage to know the exact form of the affine connection; all that is needed now is the knowledge of the commutator of the covariant derivatives which defines curvature according to a rule

$$[\nabla_\mu, \nabla_\nu]\phi^A = \mathcal{R}^A{}_{B\mu\nu}\phi^B. \tag{3}$$

The renormalized effective action constructed from the Green function of the differential operator (1) is given by

$$W_{ren} = \frac{1}{32\pi^2}\int g^{1/2}d^4x \sum_{n=3}^{\infty}\frac{(n-3)!}{(m^2)^{n-2}}\mathrm{Tr}[a_n], \tag{4}$$

where $[a_n]$ is the coincidence limit of the n-th HDSM coefficient and Tr is the matrix supertrace defined as [24]

$$\mathrm{Tr} = \mathrm{tr}_+ - \mathrm{tr}_-, \tag{5}$$

where

$$\mathrm{tr}_{\pm} f = \sum_A f_{AA}[1 \pm (-1)^{\epsilon_A}]\frac{1}{2} \tag{6}$$

and ϵ_A is the Grassman parity of ϕ^A. The coefficients a_0, a_1, and a_2 contribute to the divergent part of the action , W_{div}, which have to be absorbed into the classical gravitational action

$$S_g = \int g^{1/2} d^4x \left(\Lambda_0 + \lambda_0 R + \lambda_1 R^2 + \lambda_2 R^{\mu\nu} R_{\mu\nu} \right) + \lambda_3 \chi, \tag{7}$$

where

$$\chi = \int g^{1/2} d^4x \left(R_{\mu\mu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right), \tag{8}$$

by renormalization of the bare coupling constant. The parameters lambda should be determined experimentally and are expected to be small, since otherwise they could cause observable deviations from the predictions of the general relativity. Instead of writing out the squared Riemann tensor we used in (7) the Gauss-Bonnet invariant, which has zero functional derivative with respect to the metric tensor.

The construction of the coincidence limits of the HDSM coefficients is, except the first two, an extremely laborious task. The third coefficient, $[a_2]$, that is proportional to the anomalous trace of the renormalized stress-energy tensor of the quantized massless and conformally invariant fields has been calculated by DeWitt [12]. The coincidence limit of a_3 has been obtained by Gilkey [19,20] whereas the coefficient a_4 has been calculated by Avramidi [21-23,25] and by Amsterdamski et al. [26]. Since we are interested in the lowest order of the effective action (4) we need simple and general expression for $[a_3]$. Here we use $[a_3]$ as proposed by Avramidi [21-23] but with a different normalization:

$$[a_3] = \frac{1}{3!} \left\{ P^3 + \frac{1}{2}\{P, Z_{(2)}\} + \frac{1}{2}\left(\nabla_\mu P + \frac{1}{3} J_\mu \right) \left(\nabla^\mu P - \frac{1}{3} J^\mu \right) + \frac{1}{10} Z_{(4)} \right\}, \tag{9}$$

where

$$J_\mu = \nabla_\sigma \mathcal{R}^\sigma{}_\mu, \tag{10}$$

$$P = Q - \frac{1}{6}\hat{1} R, \tag{11}$$

$$Z_{(2)} \;=\; \Box Q \,+\, \frac{1}{2}\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} \,-\, \hat{1}\left(\frac{1}{30}R_{\mu\nu}R^{\mu\nu} \,-\, \frac{1}{30}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \,-\, \frac{1}{5}\Box R\right), \tag{12}$$

and

$$\begin{aligned} Z_{(4)} \;=\; & \Box^2 Q \,-\, \frac{1}{2}[\mathcal{R}_{\mu\nu},[\mathcal{R}^{\mu\nu},Q]] \,-\, \frac{2}{3}[J^\mu,\nabla_\mu Q] \,+\, \frac{2}{3}R^{\mu\nu}\nabla_\mu\nabla_\nu Q \,+\, \frac{1}{3}\nabla^\mu R\nabla_\mu Q \\ & +\, 2\{\mathcal{R}_{\mu\nu},\nabla^\mu J^\nu\} \,+\, \frac{8}{9}J_\mu J^\mu \,+\, \frac{4}{3}\nabla_\mu\mathcal{R}_{\rho\sigma}\nabla^\mu\mathcal{R}^{\rho\sigma} \,+\, 6\mathcal{R}_{\mu\nu}\mathcal{R}^\nu{}_\rho\mathcal{R}^{\rho\mu} \,+\, \frac{10}{3}R^{\rho\sigma}\mathcal{R}^\mu{}_\rho\mathcal{R}_{\mu\sigma} \\ & -\, R^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu}\mathcal{R}_{\rho\sigma} \,-\, \hat{1}\left(-\frac{3}{14}\Box^2 R \,-\, \frac{1}{7}R^{\mu\nu}\nabla_\mu\nabla_\nu R \,+\, \frac{2}{21}R^{\mu\nu}\Box R_{\mu\nu} \,-\, \frac{4}{7}R^\rho{}_\mu{}^\sigma{}_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu}\right. \\ & -\, \frac{4}{63}\nabla_\mu R\nabla^\mu R \,+\, \frac{1}{42}\nabla_\mu R_{\sigma\rho}\nabla^\mu R^{\sigma\rho} \,+\, \frac{1}{21}\nabla_\mu R_{\rho\sigma}\nabla^\rho R^{\sigma\mu} \,-\, \frac{3}{28}\nabla_\mu R_{\rho\sigma\lambda\tau}\nabla^\mu R^{\rho\sigma\lambda\tau} \\ & -\, \frac{2}{189}R_\mu{}^\nu R_\rho{}^\mu R_\nu{}^\rho \,+\, \frac{2}{63}R_{\rho\sigma}R^{\mu\nu}R^\rho{}_\mu{}^\sigma{}_\nu \,-\, \frac{2}{9}R_{\rho\sigma}R^\rho{}_{\mu\nu\lambda}R^{\sigma\mu\nu\lambda} \,+\, \frac{16}{189}R_{\rho\sigma}{}^{\mu\nu}R_{\mu\nu}{}^{\lambda\gamma}R_{\lambda\gamma}{}^{\rho\sigma} \\ & \left. +\, \frac{88}{189}R^\rho{}_\mu{}^\sigma{}_\nu R^\mu{}_\lambda{}^\nu{}_\gamma R^\lambda{}_\rho{}^\gamma{}_\sigma\right). \end{aligned} \tag{13}$$

In the above formulas $\hat{1}$ is the unit matrix, $\{\,,\,\}$ is the anticommutator and we have omitted the field indices.

Inserting (8) into (4) integrating by parts and making use of the elementary properties of the Riemann tensor one has

$$\begin{aligned} W_{ren} \;=\; & \frac{1}{192\pi^2 m^2}\int d^4x\, g^{1/2}\mathrm{Tr}\left\{P^3 + \frac{1}{30}P\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - R_{\mu\nu}R^{\mu\nu} + \Box R\right) + \frac{1}{2}P\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\right. \\ & +\frac{1}{2}P\Box P - \frac{1}{10}J_\mu J^\mu + \frac{1}{30}\left(2\mathcal{R}^\mu{}_\nu\mathcal{R}^\nu{}_\alpha\mathcal{R}^\alpha{}_\mu - 2R^\mu_\nu\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha\nu} + R^{\mu\nu\alpha\beta}\mathcal{R}_{\mu\nu}\mathcal{R}_{\alpha\beta}\right) \\ & +\hat{1}\left[-\frac{1}{630}R\Box R + \frac{1}{140}R_{\mu\nu}\Box R^{\mu\nu} + \frac{1}{7560}\left(-64R^\mu_\nu R^\nu_\lambda R^\lambda_\mu + 48R^{\mu\nu}R_{\alpha\beta}R^\alpha{}_\mu{}^\beta{}_\nu\right.\right. \\ & \left.\left.\left. +6R_{\mu\nu}R^\mu{}_{\alpha\beta\gamma}R^{\nu\alpha\beta\gamma} + 17R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\sigma\rho}R_{\sigma\rho}{}^{\mu\nu} - 28R^\mu{}_\alpha{}^\nu{}_\beta R^\alpha{}_\sigma{}^\beta{}_\rho R^\sigma{}_\mu{}^\rho{}_\nu\right)\right]\right\}. \end{aligned} \tag{14}$$

This first order renormalized effective action applies to any spacetime and to any differential operator of the form (1). In what follows we shall confine ourselves to the operators

$$(-\,\Box \,+\, \xi R \,+\, m^2)\phi^{(0)} \,=0, \tag{15}$$

$$(\gamma^\mu\nabla_\mu \,+\, m)\phi^{(1/2)} \;=\; 0, \tag{16}$$

$$(\delta^\mu_\nu\Box \,-\, \nabla_\nu\nabla^\mu \,-\, R^\mu_\nu \,-\, \delta^\mu_\nu m^2)\phi^{(1)} \;=\; 0, \tag{17}$$

acting on the scalar, spinor, and vector fields, respectively, where ξ is the coupling constant, and γ^μ are the Dirac matrices obeying standard relations $\gamma^\alpha\gamma^\beta + \gamma^\beta\gamma^\alpha = 2\hat{1}g^{\alpha\beta}$, and assume that the fields are neutral. Although neither (16) nor (17) has the form that allows direct application of the Schwinger-DeWitt technique the procedures described in Refs [3,24] may be used in this context. Specifically, by appropriate redefinition of the massive spinor field one obtains

$$\left(\nabla_\mu\nabla^\nu - \frac{1}{4}R - m^2\right)\phi^{(1/2)} = 0, \tag{18}$$

whereas the method presented in Refs.[24] shows that the effective action of the massive vector field is equal to the effective action of the diagonal operator

$$(\delta^\mu_\nu\Box - R^\mu_\nu - \delta^\mu_\nu m^2)\phi^{(1)} = 0, \tag{19}$$

minus the effective action of the massive scalar field with the minimal curvature coupling. The first order of the effective action is therefore

$$W^{(1)}_{ren} = \frac{1}{32\pi^2 m^2}\int g^{1/2}d^4x \begin{cases} [a_3^{(0)}] \\ -tr[a_3^{(1/2)}] \\ tr[a_3^{(1)}] - [a^{(0)}_{3|\xi=0}] \end{cases} \tag{20}$$

where the definition of the matrix supertrace has been used.

For fields obeying Eqs.(15-17) the curvature has the form

$$\mathcal{R}_{\mu\nu} = \begin{cases} 0 \\ \frac{1}{4}\gamma^\rho\gamma^\sigma R_{\rho\sigma\mu\nu} \\ R^\rho{}_{\sigma\mu\nu} \end{cases} \tag{21}$$

whereas inspection of (15), (17) and (18) shows that the potential matrix is

$$Q = \begin{cases} -\xi R \\ -\frac{1}{4}\hat{1}R \\ -R^\alpha{}_\beta \end{cases} \tag{22}$$

Inserting (21) and (22) into (14) making use of elementary properties of the Dirac matrices and Riemann tensor, after simple calculations one obtains the first term of the asymptotic expansion of the effective action in the form [21,23]

$$
\begin{aligned}
W^{(1)}_{ren} \;=\; & \frac{1}{192\pi^2 m^2}\int d^4x g^{1/2}\Big(\alpha^{(s)}_1 R\Box R \,+\, \alpha^{(s)}_2 R_{\mu\nu}\Box R^{\mu\nu} \,+\, \alpha^{(s)}_3 R^3 \,+\, \alpha^{(s)}_4 R R_{\mu\nu}R^{\mu\nu} \\
& +\, \alpha^{(s)}_5 R R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \,+\, \alpha^{(s)}_6 R^{\mu}_{\nu}R^{\nu}_{\rho}R^{\rho}_{\mu} \,+\, \alpha^{(s)}_7 R^{\mu\nu}R_{\rho\sigma}R^{\rho\ \ \sigma}_{\ \mu\ \ \nu} \\
& +\, \alpha^{(s)}_8 R_{\mu\nu}R^{\mu}_{\lambda\rho\sigma}R^{\nu\lambda\rho\sigma} \,+\, \alpha^{(s)}_9 R_{\rho\sigma}^{\ \ \ \mu\nu}R_{\mu\nu}^{\ \ \ \lambda\gamma}R_{\lambda\gamma}^{\ \ \ \rho\sigma} \,+\, \alpha^{(s)}_{10} R^{\rho\ \ \sigma}_{\ \mu\ \ \nu}R^{\mu\ \ \nu}_{\ \lambda\ \ \gamma}R^{\lambda\ \ \gamma}_{\ \rho\ \ \sigma},\Big) \\
=\; & \frac{1}{192\pi^2 m^2}\sum_{i=1}^{10}\alpha^{(s)}_i W_i,
\end{aligned}
\tag{23}
$$

where the numerical coefficients depending on the spin of the field are given in a Table I. Note that because of (20) our coefficients $\alpha^{(1/2)}_i$ for the spinor field are twice these of Avramidi's [21], and to obtain the result for the massive neutral spinor field on has to multiply $W^{(1)}_{ren}$ by the factor 1/2.

III. THE STRESS-ENERGY TENSOR IN $R = 0$ GEOMETRIES

The renormalized stress-energy tensor is given by

$$
\frac{2}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W^{(1)}_{ren} \;=\; \langle T^{\mu\nu}\rangle_{ren}, \tag{24}
$$

and for the massive scalar, spinor, and vactor fields may be rewritten in terms of the variational derivatives of the actions W_i in the form

$$
\langle T^{\mu\nu}\rangle^{(s)}_{ren} \;=\; \frac{1}{96\pi^2 m^2}\frac{1}{g^{1/2}}\sum_{i=1}^{10}\alpha^{(s)}_i\frac{\delta W_i}{\delta g_{\mu\nu}}. \tag{25}
$$

Functionally differentiating the renormalized effective action with respect to the metric tensor, performing elementary simplifications and finally retaining in the result only the terms that are nonzero for $R = 0$ geometries, after rather long calculations, one has

$$
\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_1 = (...), \tag{26}
$$

$$\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_2 = \nabla^\mu R_{\rho\sigma}\nabla^\nu R^{\rho\sigma} + \nabla^\mu R_{\rho\sigma}\nabla^\sigma R^{\rho\nu} - 3\nabla^\mu R_{\rho\sigma}\nabla^\sigma R^{\rho\nu}$$
$$+ 2\nabla^\rho\nabla^\nu\Box R_\rho{}^\mu - \Box^2 R^{\mu\nu} - \frac{1}{2}\nabla_\lambda R_{\rho\sigma}\nabla^\lambda R^{\rho\sigma}g^{\mu\nu} - \nabla^\rho\nabla^\sigma\Box R_{\rho\sigma}g^{\mu\nu}$$
$$+ 3\nabla_\sigma\nabla^\nu R_\rho{}^\mu R^{\rho\sigma} - \nabla_\sigma\nabla^\mu R_\rho{}^\nu R^{\rho\sigma} - \Box R_\rho{}^\nu R^{\rho\mu} - 3\nabla^\sigma\nabla^\mu R_{\rho\sigma}R^{\rho\nu}$$
$$- \Box R_\rho{}^\mu R^{\rho\nu} + \nabla^\rho\nabla^\nu R_{\rho\sigma}R^{\sigma\mu} + (...), \tag{27}$$

$$\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_3 = (...), \tag{28}$$

$$\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_4 = 2\nabla^\mu R_{\rho\sigma}\nabla^\nu R^{\rho\sigma} - 2\nabla_\lambda R_{\rho\sigma}\nabla^\lambda R^{\rho\sigma}g^{\mu\nu} + 2\nabla^\nu\nabla^\mu R_{\rho\sigma}R^{\rho\sigma}$$
$$- 2\Box R_{\rho\sigma}R^{\rho\sigma}g^{\mu\nu} - R_{\rho\sigma}R^{\rho\sigma}R^{\mu\nu} + (...), \tag{29}$$

$$\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_5 = 2\nabla^\mu R_{\rho\sigma\lambda\gamma}\nabla^\nu R^{\rho\sigma\lambda\gamma} - 2\nabla_\tau R_{\rho\sigma\lambda\gamma}\nabla^\tau R^{\rho\sigma\lambda\gamma}g^{\mu\nu}$$
$$+ 2\nabla^\mu\nabla^\nu R_{\rho\sigma\lambda\gamma}R^{\rho\sigma\lambda\gamma} - 2\Box R_{\rho\sigma\lambda\gamma}R^{\rho\sigma\lambda\gamma}g^{\mu\nu} - R^{\mu\nu}R_{\rho\sigma\lambda\gamma}R^{\rho\sigma\lambda\gamma} + (...), \tag{30}$$

$$\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_6 = 3\nabla^\nu R_{\rho\sigma}\nabla^\sigma R^{\rho\mu} - 3\nabla_\sigma R_\rho{}^\nu\nabla^\nu R^{\sigma\mu} - \frac{3}{2}\nabla_\lambda R_{\rho\sigma}\nabla^\sigma R^{\rho\lambda}g^{\mu\nu}$$
$$+ 3\nabla_\sigma\nabla_\nu R_\rho{}^\mu R^{\rho\sigma} - \frac{3}{2}\nabla^\sigma\nabla_\lambda R_{\rho\sigma}R^{\rho\lambda}g^{\mu\nu} + 3\nabla^\sigma\nabla^\nu R_{\rho\sigma}R^{\rho\mu} - \frac{3}{2}\Box R_\rho{}^\nu R^{\rho\mu}$$
$$- 3R_\rho{}^\sigma R_\sigma{}^\nu R^{\rho\mu} - \frac{3}{2}\Box R_\rho{}^\mu R^{\rho\nu} + \frac{1}{2}g^{\mu\nu}R_{\rho\sigma}R_\lambda{}^\rho R^{\sigma\lambda}, \tag{31}$$

$$\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_7 = \nabla_\sigma R_\rho{}^\mu\nabla^\rho R^{\sigma\nu} - 2\nabla^\nu R_{\rho\sigma}\nabla^\lambda R_\lambda{}^{\rho\sigma\mu} + 2\nabla_\lambda R_{\rho\sigma}\nabla^\nu R^{\rho\lambda\sigma\mu}$$
$$- 2\nabla_\lambda R_{\rho\sigma}\nabla^\lambda R^{\rho\mu\sigma\nu} + 2\nabla_\lambda R_{\rho\sigma}\nabla^\gamma R_\gamma{}^{\rho\sigma\lambda} - \nabla_\sigma\nabla_\rho R^{\mu\nu}R^{\rho\sigma} + 2\nabla^\lambda\nabla^\nu R_\rho{}^\mu{}_{\sigma\lambda}R^{\rho\sigma}$$
$$- \Box R_\rho{}^\mu{}_\sigma{}^\nu R^{\rho\sigma} - \nabla^\sigma\nabla^\gamma R_{\rho\sigma\lambda\gamma}R^{\rho\lambda}g^{\mu\nu} + \frac{1}{2}\nabla^\rho\nabla_\sigma R_\rho{}^\nu R^{\sigma\mu} + \frac{1}{2}\nabla^\rho\nabla_\sigma R_\rho{}^\mu R^{\sigma\nu}$$
$$+ \frac{1}{2}R_{\rho\sigma}R_{\lambda\gamma}R^{\rho\lambda\sigma\gamma} + 2\nabla_\lambda\nabla^\nu R_{\rho\sigma}R^{\rho\lambda\sigma\mu} - 3R_{\rho\sigma}R_\lambda{}^\mu R^{\rho\lambda\sigma\nu} - \nabla_\gamma\nabla_\lambda R_{\rho\sigma}R^{\rho\gamma\sigma\lambda}g^{\mu\nu}$$
$$- \Box R_{\rho\sigma}R^{\rho\mu\sigma\nu}, \tag{32}$$

$$\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}W_8 = -2\nabla_\sigma R_\rho{}^\mu\nabla^\lambda R_\lambda{}^{\rho\sigma\nu} + \nabla_\sigma R_\rho{}^\mu\nabla^\lambda R_\lambda{}^{\nu\rho\sigma}$$
$$+ \nabla^\nu R_{\rho\sigma\lambda\gamma}\nabla^\gamma R^{\rho\sigma\lambda\mu} - \nabla_\gamma R_{\rho\sigma\lambda}{}^\mu\nabla^\gamma R^{\rho\sigma\lambda\mu} + \nabla^\sigma R_{\rho\sigma\lambda\gamma}\nabla^\nu R^{\rho\mu\lambda\gamma}$$
$$- 2\nabla_\lambda R_{\rho\sigma}\nabla^\sigma R^{\rho\mu\lambda\nu} - \frac{1}{2}\nabla^\rho R_{\rho\sigma\lambda\gamma}\nabla^\tau R_\tau{}^{\sigma\lambda\sigma}g^{\mu\nu} + \frac{1}{2}\nabla_\tau R_{\rho\sigma\lambda\gamma}\nabla^\lambda R^{\rho\sigma\gamma\tau}g^{\mu\nu}$$

$$
\begin{aligned}
&- 2\nabla_\lambda \nabla^\rho R_{\rho\ \sigma}^{\ \nu\ \mu} R^{\sigma\lambda} - 2\nabla^\rho \nabla^\lambda R_{\rho\sigma\lambda}^{\quad\ \nu} R^{\sigma\mu} + \nabla^\nu \nabla_\gamma R_{\rho\ \sigma\lambda}^{\ \mu} R^{\rho\gamma\sigma\lambda} \\
&+ R_\rho^{\ \mu} R_{\sigma\lambda\gamma}^{\quad\ \nu} R^{\rho\gamma\sigma\lambda} - \frac{1}{2}\nabla_\tau \nabla^\sigma R_{\rho\sigma\lambda\gamma} R^{\rho\tau\lambda\gamma} g^{\mu\nu} - \frac{1}{2}\Box R_{\rho\ \sigma\lambda}^{\ \nu} R^{\rho\mu\sigma\lambda} \\
&+ \nabla^\sigma \nabla^\nu R_{\rho\sigma\lambda\gamma} R^{\rho\mu\lambda\gamma} - \frac{1}{2}\Box R_{\rho\ \sigma\lambda}^{\ \mu} R^{\rho\nu\sigma\lambda} + 2R_{\rho\sigma} R_{\lambda\ \gamma}^{\ \rho\ \mu} R^{\sigma\lambda\gamma\nu} \\
&+ \frac{1}{2}\nabla_\tau \nabla^\rho R_{\rho\sigma\lambda\gamma} R^{\sigma\tau\lambda\gamma} g^{\mu\nu} - \frac{1}{2} R_{\rho\sigma} R_{\lambda\gamma\tau}^{\quad\ \rho} R^{\sigma\tau\lambda\gamma} - 2\nabla^\rho \nabla_\lambda R_{\rho\sigma} R^{\sigma\mu\lambda\nu} \\
&+ \nabla_\lambda \nabla_\sigma R_\rho^{\ \mu} R^{\rho\lambda\sigma\nu},
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}} W_9 = &-6\nabla^\lambda R_{\rho\sigma\lambda}^{\quad\ \nu} \nabla^\gamma R_\gamma^{\ \mu\rho\sigma} - 6\nabla_\gamma R_{\rho\sigma\lambda}^{\quad\ \nu} \nabla^\lambda R^{\rho\sigma\gamma\mu} \\
&- 3R_{\rho\sigma\lambda}^{\quad\ \mu} R_{\gamma\tau}^{\ \ \lambda\nu} R^{\rho\sigma\gamma\tau} - 2\nabla^\lambda \nabla_\gamma R_{\rho\sigma\lambda}^{\quad\ \nu} R^{\rho\sigma\gamma\mu} - 4\nabla^\lambda \nabla_\gamma R_{\rho\sigma\lambda}^{\quad\ \mu} R^{\rho\sigma\gamma\nu} \\
&- 4\nabla_\gamma \nabla^\rho R_{\rho\ \sigma\lambda}^{\ \mu} R^{\sigma\lambda\gamma\mu} - 2\nabla_\gamma \nabla^\rho R_{\rho\ \sigma\lambda}^{\ \mu} R^{\sigma\lambda\gamma\nu} + \frac{1}{2} R_{\rho\sigma\lambda\gamma} R_{\tau\kappa}^{\ \ \rho\sigma} R^{\lambda\gamma\tau\kappa} g^{\mu\nu},
\end{aligned}
\tag{34}
$$

and

$$
\begin{aligned}
\frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}} W_{10} = &\ 3\nabla^\rho R_{\rho\sigma\lambda}^{\quad\ \mu} \nabla^\gamma R_\gamma^{\ \lambda\sigma\nu} + 3\nabla_\gamma R_{\rho\sigma\lambda}^{\quad\ \nu} \nabla^\sigma R^{\rho\mu\lambda\gamma} \\
&+ 3\nabla^\rho R_{\rho\sigma\lambda\gamma} \nabla^\gamma R^{\sigma\mu\lambda\nu} + 3\nabla^\rho R_{\rho\sigma\lambda\gamma} \nabla^\gamma R^{\sigma\nu\lambda\mu} - 3\nabla_\gamma \nabla_\lambda R_{\rho\ \sigma}^{\ \mu\ \nu} R^{\rho\gamma\sigma\lambda} \\
&+ \nabla_\gamma \nabla^\lambda R_{\rho\ \sigma\lambda}^{\ \nu} R^{\rho\gamma\sigma\mu} + 2\nabla_\gamma \nabla^\lambda R_{\rho\ \sigma\lambda}^{\ \mu} R^{\rho\gamma\sigma\nu} + 3R_{\rho\sigma\lambda\gamma} R_\tau^{\ \sigma\gamma\mu} R^{\rho\tau\lambda\nu} \\
&+ 2\nabla^\sigma \nabla_\gamma R_{\rho\sigma\lambda}^{\quad\ \nu} R^{\rho\mu\lambda\gamma} - 3\nabla^\sigma \nabla^\gamma R_{\rho\sigma\lambda\gamma} R^{\rho\mu\lambda\nu} + \nabla^\sigma \nabla_\gamma R_{\rho\sigma\lambda}^{\quad\ \mu} R^{\rho\nu\lambda\gamma} \\
&- \frac{1}{2} R_{\rho\sigma\lambda\gamma} R_{\tau\ \kappa}^{\ \rho\ \gamma} R^{\sigma\tau\lambda\kappa} g^{\mu\nu},
\end{aligned}
\tag{35}
$$

where the ellipsis denote omitted terms that contain the scalar curvature and its covariant derivatives. For nondiagonal $R = 0$ metrics obtained $\langle T_\nu^\mu \rangle_{ren}^{(s)}$ must be symmetrized. As expected, the functional derivatives W_1 and W_3 do not contribute to the stress-energy tensor in the Reissner-Nordström spacetime. It should be noted that Eqs (26-35) have been obtained by putting $R = 0$ in the general result, which is more complex and shall not be presented here. Inspection of Eqs. (26-35) shows that to construct the stress-energy tensor of the massive fields in the Ricci-flat geometry it suffices to analyse only W_5, W_8, W_9, W_{10}, and therefore our $\langle T_\nu^\mu \rangle_{ren}^{(s)}$ generalizes earlier results derived by Frolov and Zel'nikov [1-3].

IV. $\langle T_\nu^\mu\rangle_{REN}^{(S)}$ IN THE REISSNER-NORDSTRÖM SPACETIME

Now we are ready to construct the stress-energy tensor of the massive quantized fields in the nonextremal Reissner-Nordström geometry described by the line element

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 + r^2 \left(\sin^2\theta d\phi^2 + d\theta^2\right), \quad (36)$$

where M is the mass and e is a charge of the black hole. For $e^2 < M^2$ the equation $g_{00} = 0$ has two positive roots

$$r_\pm = M \pm (M^2 - e^2)^{1/2}, \quad (37)$$

and the larger root represents the location of the event horizon whereas r_- is the inner horizon. In the limit $e^2 = M^2$ horizons merge at $r = M$, and the the Riessner-Nordström solution degenerates to the extremal one with the line element given by

$$ds^2 = \left(1 - \frac{M}{r}\right)^2 dt^2 + \left(1 - \frac{M}{r}\right)^{-2} dr^2 + r^2 \left(\sin^2\theta d\phi^2 + d\theta^2\right). \quad (38)$$

Although the stress-energy tensor could be evaluated in the nonstatic background (provided the changes of the geometry are slow) we shall confine ourselves to the exterior region where the spacetime is static. We begin with the massive scalar field extensively studied in the Ref.[8]. Constructing components of the Riemann tensor, its contractions and covariant derivatives and subsequently inserting them with appropriate coefficients $\alpha_i^{(0)}$ into (25) we have

$$\langle T_\nu^\mu\rangle_{ren}^{(0)} = C_\nu^\mu + \left(\xi - \frac{1}{6}\right) D_\nu^\mu, \quad (39)$$

where

$$C_t^t = -\frac{1}{30240\,\pi^2 m^2 r^{12}} \Big(1248\,e^6 - 810\,r^4 e^2 + 855\,M^2 r^4 + 202\,r^2 e^4 - 1878\,M^3 r^3 + 1152\,M r^3 e^2 + 2307\,M^2 r^2 e^2 - 3084\,r M e^4\Big), \quad (40)$$

$$D_t^t = \frac{1}{720\,\pi^2 m^2 r^{12}} \Big(-792\,M^3 r^3 + 360\,M^2 r^4 + 2604\,M^2 r^2 e^2 - 1008\,M r^3 e^2 - 2712\,r M e^4 + 819\,e^6 + 728\,r^2 e^4\Big), \quad (41)$$

$$C_r^r = \frac{1}{30240\,\pi^2\,m^2\,r^{12}}\Big(444\,e^6 - 1488\,Mr^3e^2 + 162\,r^4e^2 + 842\,r^2e^4 - 1932\,rMe^4$$
$$+\ 315\,M^2r^4 + 2127\,M^2r^2e^2 - 462\,M^3r^3\Big)\,, \tag{42}$$

$$D_r^r = \frac{1}{720\,\pi^2\,m^2\,r^{12}}\Big(-792\,M^3r^3 + 360\,M^2r^4 + 2604\,M^2r^2e^2 - 1008\,Mr^3e^2$$
$$-\ 2712\,rMe^4 + 819\,e^6 + 728\,r^2e^4\Big)\,, \tag{43}$$

$$C_\theta^\theta = -\frac{1}{30240\,\pi^2\,m^2\,r^{12}}\Big(3044\,r^2e^4 - 2202\,M^3r^3 - 10356\,rMe^4$$
$$+\ 3066\,e^6 - 4884\,r^3Me^2 + 9909\,r^2M^2e^2 + 945\,M^2r^4 + 486\,r^4e^2\Big)\,, \tag{44}$$

and

$$D_\theta^\theta = \frac{1}{720\,\pi^2\,m^2\,r^{12}}\Big(3276\,r^2M^2e^2 - 1176\,r^3Me^2 - 3408\,rMe^4 + 1053\,e^6$$
$$-\ 1008\,M^3r^3 + 432\,M^2r^4 + 832\,r^2e^4\Big)\,. \tag{45}$$

Obtained result for nonvanishing components of the stress-energy tensor coincides with the $\langle T_\nu^\mu\rangle_{ren}$ constructed by Anderson, Hiscock and Samuel. This coincidence is, of course, not surprising as there is a one-to-one correspondence between the order of the WKB approximation and the order of the Schwinger-DeWitt expansion. To obtain the $m^{-2}-$terms one has to use 6-th order WKB approximation of the mode functions and the results (39-45) are simply manifestation of this correspondence.

Having computed functional derivatives of W_i with respect to the metric tensor the construction of the stress-energy tensor of the massive fields of higher spins present no problems. Indeed, inserting coefficients $\alpha_i^{(1/2)}$ for the neutral spinor field into (25) one obtains

$$\langle T_t^t\rangle_{ren}^{(1/2)} = \frac{1}{40320\,\pi^2\,m^2\,r^{12}}\Big(2384\,M^3r^3 + 10544\,r^2e^4 - 22464\,r^3Me^2 + 21832\,r^2M^2e^2$$
$$-\ 1080\,M^2r^4 - 21496\,rMe^4 + 4917\,e^6 + 5400\,r^4e^2\Big)\,, \tag{46}$$

$$\langle T_r^r\rangle_{ren}^{(1/2)} = \frac{1}{40320\pi^2\,m^2\,r^{12}}\Big(504\,M^2r^4 + 1080\,r^4e^2 - 784\,M^3r^3 - 6336\,r^3Me^2$$
$$+\ 3560\,r^2e^4 + 8440\,r^2M^2e^2 - 8680\,rMe^4 + 2253\,e^6\Big)\,, \tag{47}$$

and

$$\langle T_\theta^\theta\rangle_{ren}^{(1/2)} = -\frac{1}{40320\,\pi^2\,m^2\,r^{12}}\Big(-3536\,M^3r^3 + 12080\,r^2e^4 - 20016\,r^3Me^2 + 30808\,r^2M^2e^2 + 1512\,M^2r^4 - 33984\,rMe^4 + 9933\,e^6 + 3240\,r^4e^2\Big). \tag{48}$$

Similarly, repeating the steps for the massive vector field one gets

$$\langle T_t^t\rangle_{ren}^{(1)} = -\frac{1}{10080\,\pi^2\,m^2\,r^{12}}\Big(-31057\,e^6 - 1665\,M^2r^4 - 41854\,r^2e^4 - 93537\,r^2e^2M^2 + 107516\,rMe^4 + 3666\,M^3r^3 + 69024\,r^3e^2M - 12150\,e^2r^4\Big), \tag{49}$$

$$\langle T_r^r\rangle_{ren}^{(1)} = \frac{1}{10080\,\pi^2\,m^2\,r^{12}}\Big(1050\,M^3r^3 - 693\,M^2r^4 + 12907\,r^2e^2M^2 - 10448\,r^3e^2M - 16996\,rMe^4 + 2430\,e^2r^4 + 6442\,r^2e^4 + 5365\,e^6\Big), \tag{50}$$

and

$$\langle T_\theta^\theta\rangle_{ren}^{(1)} = -\frac{1}{10080\,\pi^2\,m^2\,r^{12}}\Big(13979\,e^6 - 2079\,M^2r^4 + 20908\,r^2e^4 + 30881\,r^2e^2M^2 - 44068\,rMe^4 + 4854\,m^3r^3 - 31708\,r^3e^2M + 7290\,e^2r^4\Big). \tag{51}$$

Simple calculations show that the tensors (46-48) and (49-51) are covariantly conserved. Moreover, it could be easily verified that taking the limit $e = 0$ results, as expected, in the formulas derived by Frolov and Zel'nikov in the Schwarzschild spacetime (see for example Ref. [5]). Although there are no numeric calculations of the stress-energy tensor of the massive spinor and vector fields against which one could test the results (46-51), we expect that the approximation is reasonable so long the mass of the field is sufficiently large.

Since the Schwinger-DeWitt approximation is local and the geometry at the event horizon is regular, one expects that the stress-energy tensor is also regular there. Indeed, it could be easily shown that if there are no fluxes of energy the regularity conditions on the event horizon [27,28] require that the components of the stress-energy tensor and

$$\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1/2}\left(\langle T_t^t\rangle_{ren}^{(0)} - \langle T_r^r\rangle_{ren}^{(0)}\right) \tag{52}$$

remain finite as $r \to r_+$. Since the difference between the time and radial components of the stress-energy tensor factors, i.e.

$$\langle T_t^t \rangle_{ren}^{(s)} - \langle T_r^r \rangle_{ren}^{(s)} = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) F^{(s)}(r), \tag{53}$$

where $F^{(s)}(r)$ for each spin of the field is a simple polynomial in $1/r$, one can draw a general conclusion that our approximate the stress-energy tensors are regular as one approaches the event horizon. Analyses carried out in Ref.[8] indicate that all components of the numerically evaluated stress-energy tensor of the massive scalar field are also finite on the event horizon. Repeating calculations in the spacetime of the extreme Reissner-Nordström black hole, one obtains the stress-energy tensor that is regular on the event horizon. Simple calculations show that

$$\left(\langle T_t^t \rangle_{ren}^{(s)} - \langle T_r^r \rangle_{ren}^{(s)}\right) \left(1 - \frac{M}{r}\right)^{-2} \tag{54}$$

is finite at $r = M$.

To study obtained $\langle T_\mu^\nu \rangle_{ren}^{(s)}$ further it is useful to introduce new coordinate $x = (r - r_+)/M$ and a new parameter $q = |e|/M$. Since for the massive scalar field Anderson, Hiscock, and Samuel have found that for $m \geq 2/M$ the Schwinger-DeWitt approximation is rather good near the event horizon we also take this bound in our calculations of the vacuum polarization of the spinor and vector fields. Our results for $q = 0$, $q = 0.95$, and $m\,M = 2$ are displayed in the Figures 1-6. Inspection of the figures shows that for q close to the extremal value, the radial dependence of the components of the stress-energy tensor of the massive spinor field and their vector counterparts is qualitatively similar. On the other hand, for small q only the radial components exhibits such a similarity. Indeed, on the event horizon $\langle T_t^t \rangle_{ren}^{(1)}$ and $\langle T_t^t \rangle_{ren}^{(1/2)}$ differ in sign. The difference in the sign of the horizon values of the stress-energy tensor occurs also for the tangential components. Moreover, in the vicinity of the event horizon the magnitude of the vacuum polariation effects increases with spin of the quantized field.

Geometries that could be obtained from nonextremal black holes taking the extremality limit and expanding the near-horizon geometry into the whole manifold received recently

much attention. Near the event horizon of the extremal Reissner-Nordström black hole the geometry approaches that of the Bertotti-Robinson [29]

$$ds^2 = \frac{M^2}{\tilde{r}^2}\left(-dt^2 + d\tilde{r}^2 + \tilde{r}^2 d\theta^2 + \tilde{r}^2 \sin^2\theta d\phi^2\right), \tag{55}$$

as could be demonstrated [28] expanding the line element (55) in power series about the event horizon and subsequently making substitution

$$r = M\left(1 + \frac{M}{\tilde{r}}\right). \tag{56}$$

The stress-energy tensor of the massive fields in the Bertotti-Robinson may be easily obtained either by constructing the curvature terms for the line element (55) and inserting them into (25) or taking $|e| = M$ limit in the $\langle T_{\nu}^{\mu}\rangle_{ren}^{(s)}$ near the event horizon. Simple calculations give [30]

$$\langle T_{\nu}^{\mu}\rangle_{ren}^{(s)} = \frac{\mu^{(s)}}{2880\pi^2 m^2 M^6}\mathrm{diag}[1, 1, -1, -1], \tag{57}$$

where

$$\mu^{(s)} = \begin{cases} \frac{16}{21} - 4(\xi - \frac{1}{6}) \\ \frac{37}{14} \\ \frac{114}{7} \end{cases}. \tag{58}$$

Assuming that the renormalized cosmological constant, Λ_{ren}, is zero in the analog of the gravitational action (7) with the renormalized bare lambda coefficients, the Bertotti-Robinson geometry is a self-consistent solution of the semiclassical Einstein equations with the source term given by the stress-energy tensor of the massive field in the large mass limit [6] if $\mu^{(s)} < 0$. It is because

$$H^{\mu\nu} = \frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}\int d^4x g^{1/2} R^2, \tag{59}$$

and

$$I^{\mu\nu} = \frac{1}{g^{1/2}}\frac{\delta}{\delta g_{\mu\nu}}\int d^4x g^{1/2} R_{\mu\nu}R^{\mu\nu}, \tag{60}$$

vanish for the line element (55). An interesting consequence of (57) is that a self-consistent solution is possible for the massive scalar field provided $\xi > 5/14$, whereas for massive spinor and vector fields appropriate solutions do not exist. It should be noted however, that the stress-energy tensor of the massive scalar field with the physically most plausible values of the coupling constant, namely $\xi = 0$ and $\xi = 1/6$, do not yield self-consistent solutions and therefore the scalar field is not different than the spinor or vector field in this regard.

V. CONCLUDING REMARKS

In this paper we have constructed the renormalized stress-energy tensor of the massive scalar, spinor, and vector fields in the Reissner-Nordström spacetime. The method employed here is based on observation that the first order effective action could be expressed in terms of the traced coincidence limit of the coefficient a_3. The general $\langle T^{\mu}_{\nu}\rangle^{(s)}_{ren}$, that has been obtained by functional differentiation of the effective action with respect to a metric tensor, consists of over one hundred terms, such as the terms cubic in curvature or involving fourth derivatives. Since even after simplifications, the final result is rather complicated the specific calculations are long but straightforward.

Applying Eqs.(26-35) to the massive scalar field we rederived the results of Anderson, Hiscock, and Samuel. Their calculations were based on the WKB approximation of the solutions of the scalar field equation and summation thus obtained mode functions by means of the Abel-Plana formula. On the other hand, the method employed here may be regarded as geometrical and the identity of results is, although expected, impressive. To our knowledge spinor and vector fields have not been discussed earlier.

The results (39-51) have also been used to construct and analyse stress-energy tensor in the two interesting limiting cases that could be obtained from the Reissner-Nordström solution: the extremal Reissner-Nordström and Bertotti-Robinson geometries. Because of the form of the stress-energy tensor and the fact that the variational derivatives of the functionals constructed from $R_{\mu\nu}R^{\mu\nu}$ and R^2 vanish in the Bertotti-Robinson spacetime,

this geometry may be a self-consistent solution of the semiclassical Einstein field equations. We found that the self-consistent solution is possible for the massive scalar field provided $\xi > 5/14$, whereas for massive spinor and vector fields such solutions do not exist.

Finally, we remark that it would be interesting to construct the next order of the renormalized effective action (4). As the functional W_{ren} at that order involves coincidence limit of the a_4 coefficient, which is, in turn, given by a very complicated formula, one expect that such a calculation would be a real challenge. Another important direction of investigation is generalization of the obtained results to the elliptic operators (1) with other physically interesting matrix potentials Q and curvatures $\mathcal{R}^{A}{}_{B}$, and to analyse the back reaction of the quantized massive fields on the metric. We hope that the results obtained in this paper will be of use in further calculations.

It should be emphasized however, that being local in its nature, the Schwinger-DeWitt expansion does not describe particle creation which is a nonperturbative and nonlocal phenomenon. Moreover, in the massless limit the method breaks down. To address successfully this group of problems new methods are necessary as, for example, the covariant perturbation theory [31].

TABLES

TABLE I. The coefficients $\alpha_i^{(s)}$ for the massive scalar, spinor, and vector field

	s = 0	s = 1/2	s = 1
$\alpha_1^{(s)}$	$\frac{1}{2}\xi^2 - \frac{1}{5}\xi + \frac{1}{56}$	$-\frac{3}{140}$	$-\frac{27}{280}$
$\alpha_2^{(s)}$	$\frac{1}{140}$	$\frac{1}{14}$	$\frac{9}{28}$
$\alpha_3^{(s)}$	$\left(\frac{1}{6} - \xi\right)^3$	$\frac{1}{432}$	$-\frac{5}{72}$
$\alpha_4^{(s)}$	$-\frac{1}{30}\left(\frac{1}{6} - \xi\right)$	$-\frac{1}{90}$	$\frac{31}{60}$
$\alpha_5^{(s)}$	$\frac{1}{30}\left(\frac{1}{6} - \xi\right)$	$-\frac{7}{720}$	$-\frac{1}{10}$
$\alpha_6^{(s)}$	$-\frac{8}{945}$	$-\frac{25}{376}$	$-\frac{52}{63}$
$\alpha_7^{(s)}$	$\frac{2}{315}$	$\frac{47}{630}$	$-\frac{19}{105}$
$\alpha_8^{(s)}$	$\frac{1}{1260}$	$\frac{19}{630}$	$\frac{61}{140}$
$\alpha_9^{(s)}$	$\frac{17}{7560}$	$\frac{29}{3780}$	$-\frac{67}{2520}$
$\alpha_{10}^{(s)}$	$-\frac{1}{270}$	$-\frac{1}{54}$	$\frac{1}{18}$

REFERENCES

[1] V. P. Frolov and A. I. Zel'nikov, Phys. Lett. B **115**, 372 (1982)

[2] V. P. Frolov and A. I. Zel'nikov, Phys. Lett. B **123**, 197 (1983)

[3] V. P. Frolov and A. I. Zel'nikov, Phys. Rev. D **29**, 1057 (1984)

[4] V. P. Frolov, Proceedings of the Lebedev Institute of the Academy of Science of USSR **169** (1986)

[5] V. P. Frolov and I. D. Novikov, *Black Hole Physics* (Kluwer Dordrecht 1998)

[6] L. A. Kofman and V. Sahni, Phys. Lett. B **127**, 197 (1983)

[7] P. R. Anderson, Phys. Rev. D **41**, 1152 (1990)

[8] P. R. Anderson, W. A. Hiscock, and D. A. Samuel, Phys. Rev. D **51**, 4337 (1995)

[9] W. A. Hiscock, S. L. Larson, and P. R. Anderson, Phys. Rev. D **56**, 3571 (1997)

[10] B. E. Taylor, W. A. Hiscock, and P. R. Anderson, Phys. Rev. D **55**, 6116 (1997)

[11] R. Herman, Phys. Rev. D **58**, 084028 (1998)

[12] B. S. DeWitt, *Dynamical theory of groups and fields* (Gordon and Breach, New York 1965)

[13] S. M. Christensen, Phys. Rev. D **14**, 2490 (1976)

[14] S. M. Christensen, Phys. Rev. D **17**, 946 (1978)

[15] B. S. DeWitt, Phys. Reports **19 C**, 297 (1975)

[16] J. S. Schwinger, Phys. Rev. **82**, 664 (1951)

[17] Birrell and P. C. W. Davies, *Quantum fields in curved space* (Cambridge University Press, Cambridge 1984)

[18] S. A. Fulling, *Aspect of quantum field theory in curved space-time* (Cambridge University Press, Cambridge 1989)

[19] P. B. Gilkey, J. Diff. Geom, **10**, 601 (1975)

[20] P. B. Gilkey, Trans. Am. Math. Soc. **225**, 341 (1977)

[21] I. G. Avramidi, hep-th/9510140

[22] I. G. Avramidi, Teor. Mat. Fiz, **79**, 219 (1989)

[23] I. G. Avramidi, Nucl. Phys. **B 355**, 712 (1991)

[24] A. O. Barvinsky and G. A. Vilkovisky, Phys. Reports **119**, 1 (1985)

[25] I. G. Avramidi, Phys. Lett. B **238**, 92 (1990)

[26] P. Amsterdamski, A. L. Berkin, and D. J. O'Connor, Class. Quantum Grav. **6**, 1981 (1989)

[27] C.-G. Huang, Phys. Lett. A **164**, 384 (1992)

[28] P. R. Anderson, W. A. Hiscock, and D. J. Loranz, Phys. Rev. Lett. **74**, 4365 (1995)

[29] B. Bertotti, Phys. Rev. **116**, 1331 (1959); I. Robinson, Bull. Acad. Pol. Sci. **7**, 351 (1959)

[30] Our $\mu^{(0)}$ differs from the value evaluated in Ref. [6] which is insenstive to the coupling constant ξ.

[31] G. A. Vilkovisky, in *Quantum Theory of Gravity*, edited by S. M. Christensen (Hilger, Bristol, 1984).

FIGURES

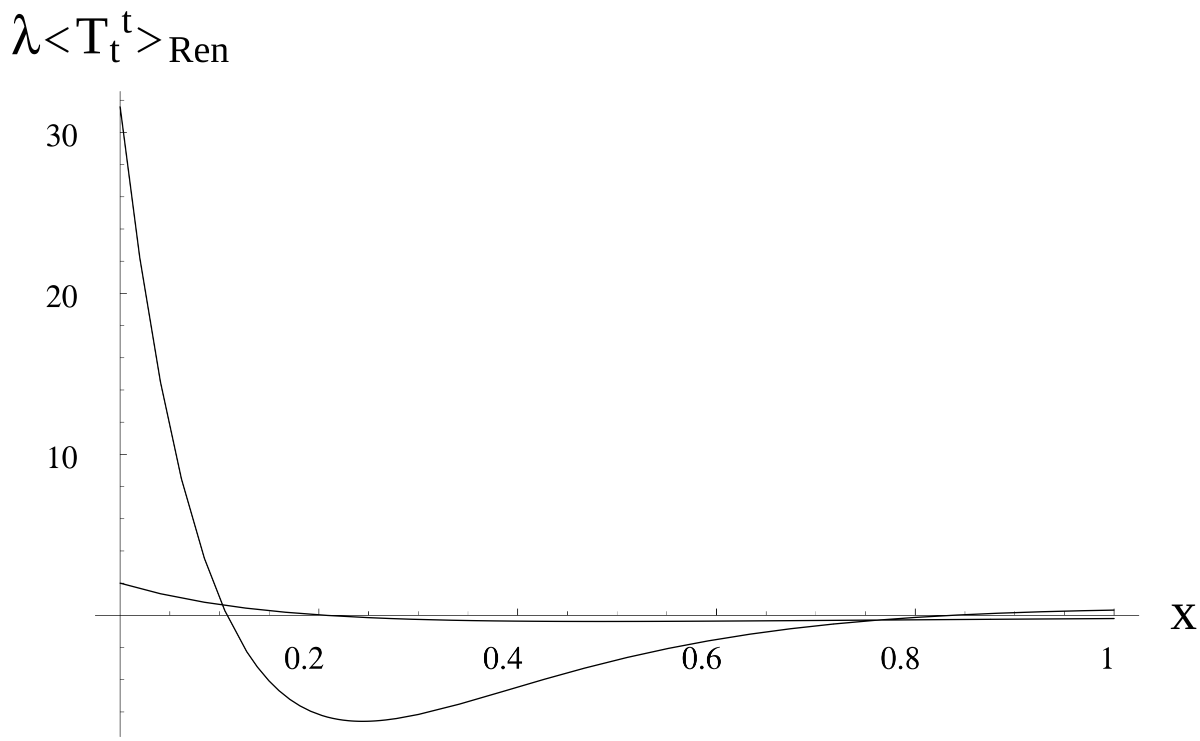

FIG. 1. This graph shows the radial dependence of the rescaled component $\langle T_t^t \rangle_{ren}^{(1/2)}$ ($\lambda = 180(8M)^4\pi^2$) of the renormalized stress-energy tensor of the massive spinor field with $m = 2/M$. From top to bottom the cureves are for $q = 0.95$ and $q = 0$.

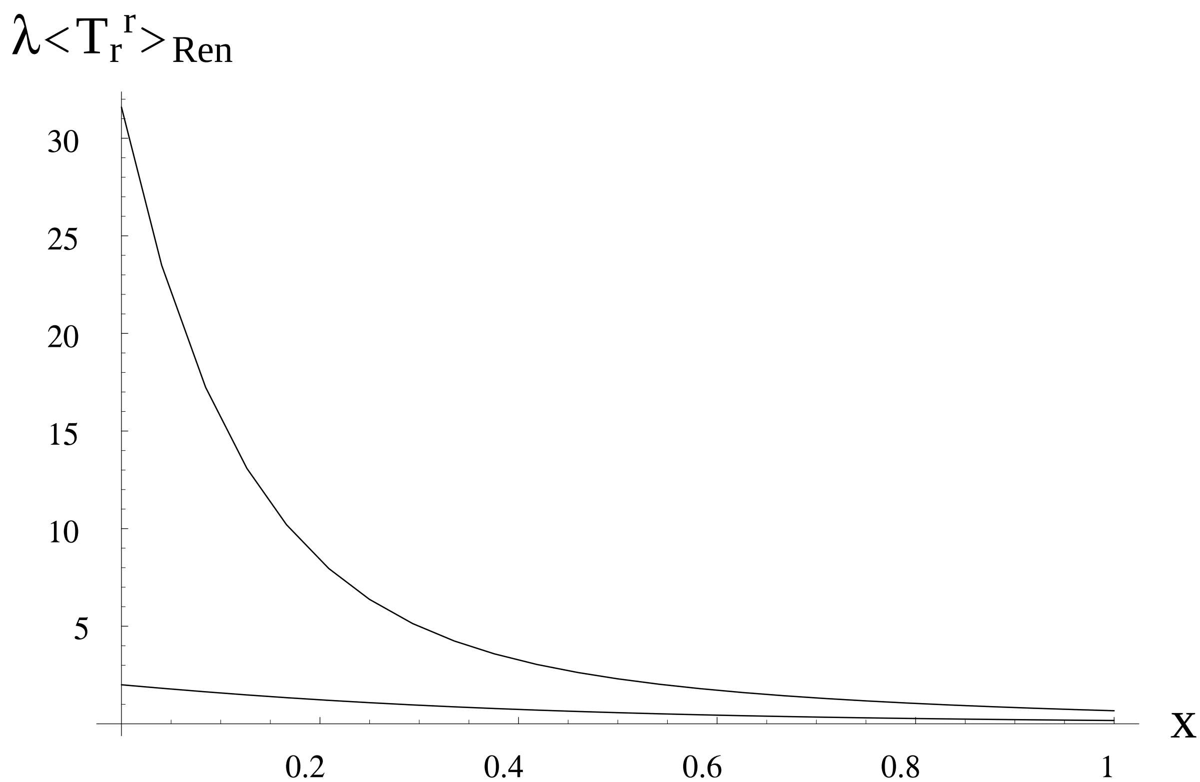

FIG. 2. This graph shows the radial dependence of the rescaled component $\langle T_r^r \rangle_{ren}^{(1/2)}$ ($\lambda = 180(8M)^4\pi^2$) of the renormalized stress-energy tensor of the massive spinor field with $m = 2/M$. From top to bottom the cureves are for $q = 0.95$ and $q = 0$.

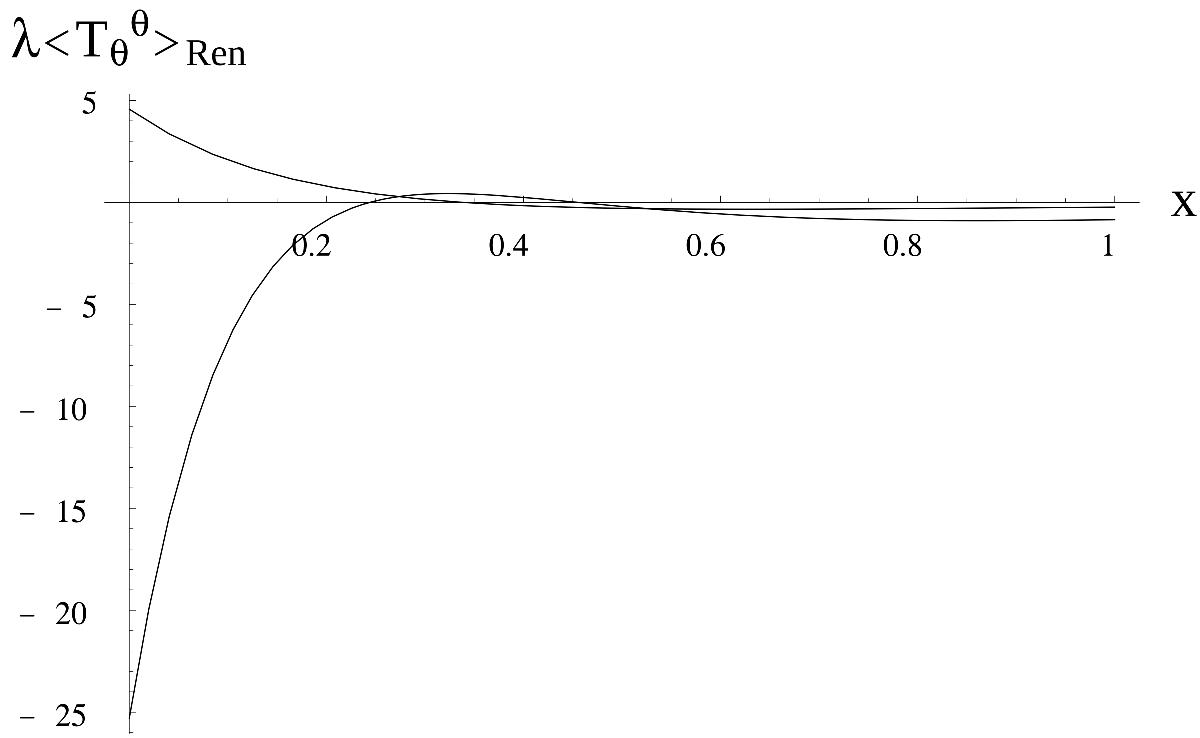

FIG. 3. This graph shows the radial dependence of the rescaled component $\langle T_\theta^\theta\rangle_{ren}^{(1/2)}$ ($\lambda = 180(8M)^4\pi^2$) of the renormalized stress-energy tensor of the massive spinor field with $m = 2/M$. From top to bottom the cureves are for $q = 0$ and $q = 0.95$

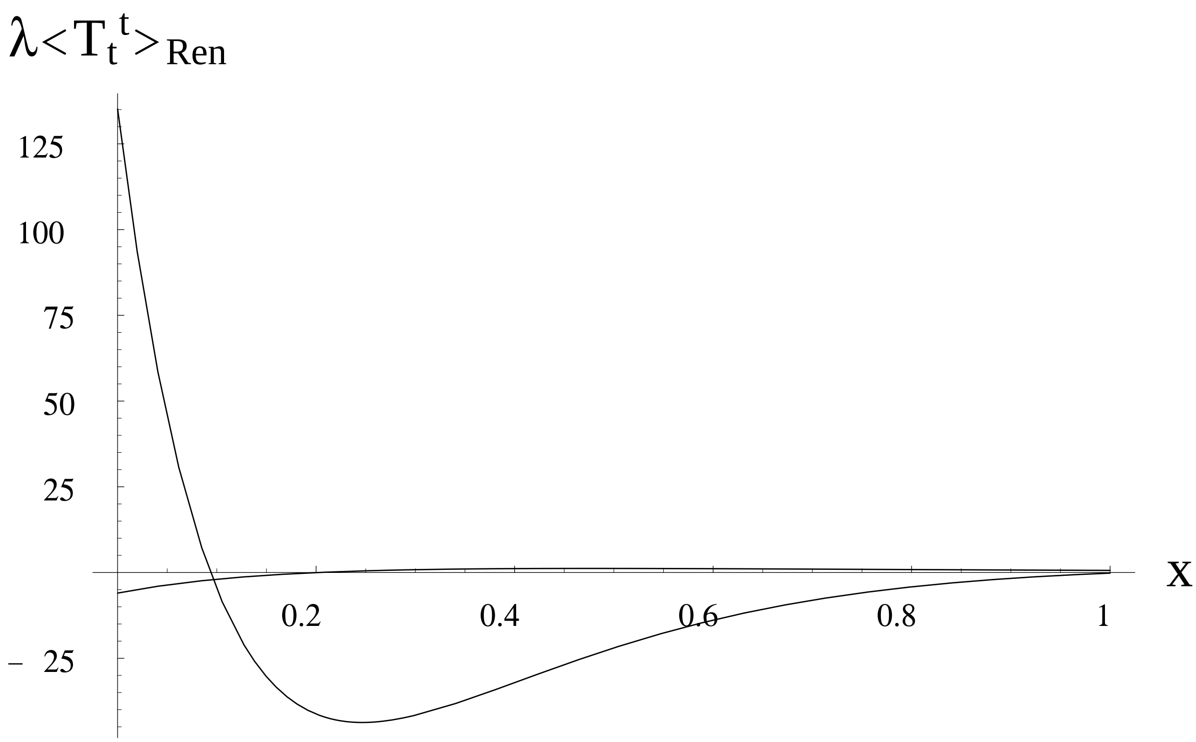

FIG. 4. This graph shows the radial dependence of the rescaled component $\langle T_t^t \rangle_{ren}^{(1)}$ ($\lambda = 90(8M)^4\pi^2$) of the renormalized stress-energy tensor of the massive vector field with $m = 2/M$. From top to bottom the cureves are for $q = 0.95$ and $q = 0$.

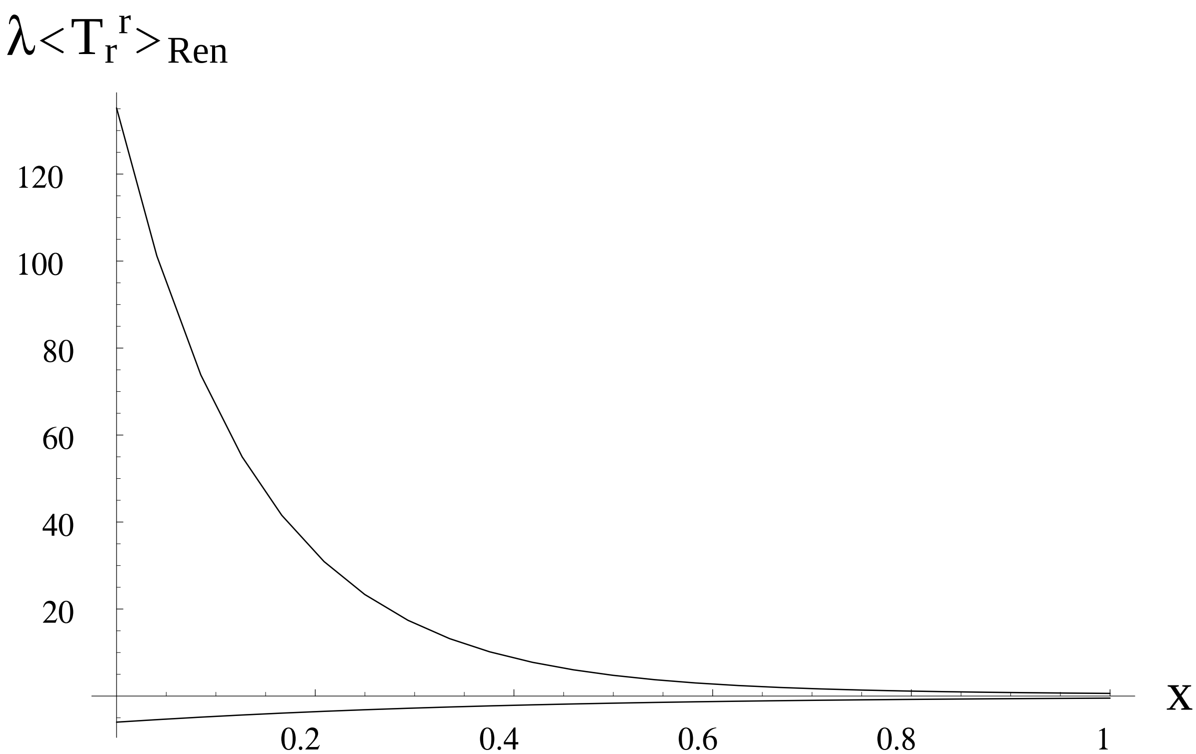

FIG. 5. This graph shows the radial dependence of the rescaled component $\langle T_r^r \rangle_{ren}^{(1)}$ ($\lambda = 90(8M)^4\pi^2$) of the renormalized stress-energy tensor of the massive vector field with $m = 2/M$. From top to bottom the cureves are for $q = 0.95$ and $q = 0$.

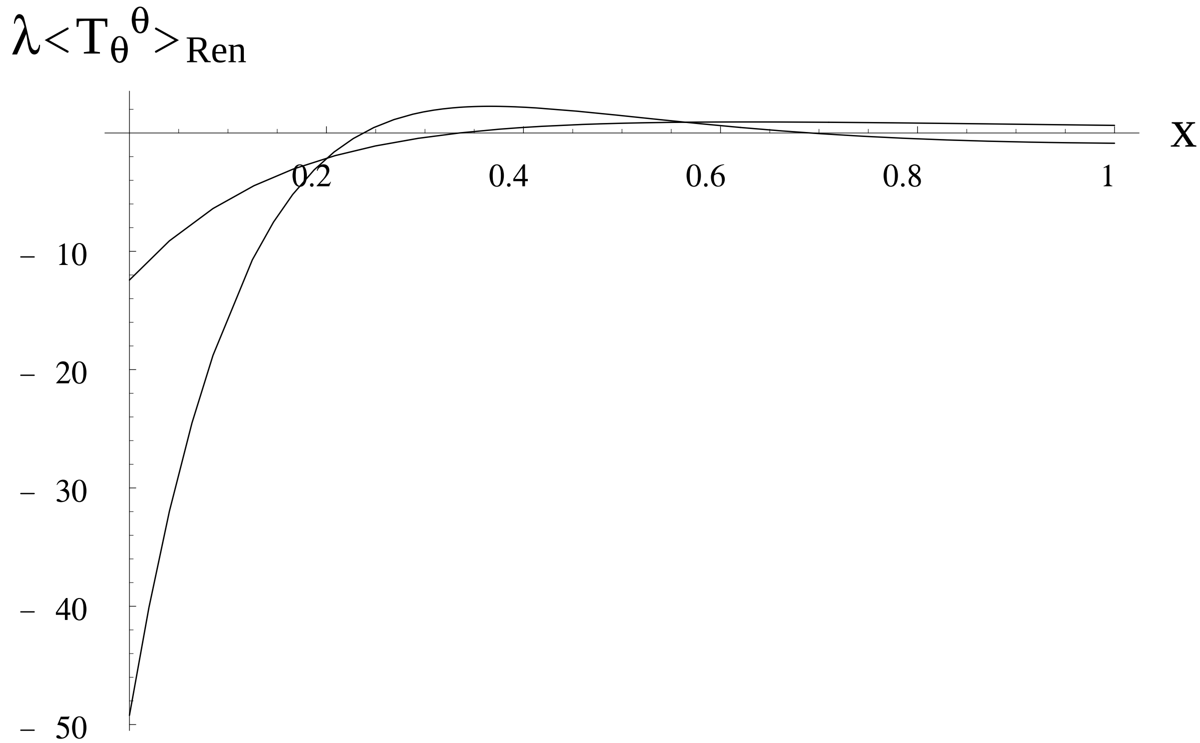

FIG. 6. This graph shows the radial dependence of the rescaled component $\langle T_\theta^\theta\rangle_{ren}^{(1)}$ ($\lambda = 90(8M)^4\pi^2$) of the renormalized stress-energy tensor of the massive vector field with $m = 2/M$. From top to bottom the cureves are for $q = 0$ and $q = 0.95$